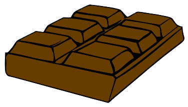
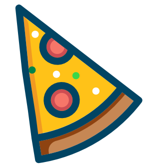


CEMC Math Circles - Grade 11/12

November 18 - 24, 2020
Partitioning Puzzles



This week, we're going to look at some very different problems. Once again, they don't require "typical" high school mathematics. You can try to work them out with brute force methods, but more often than not there is a clever solution waiting to be found. The only common theme: they all have to do with dividing objects into smaller pieces. The problems are easy to understand, so have a look at each one and think about them throughout the week.

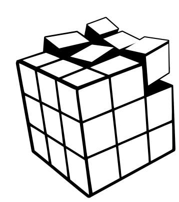
Disclaimer: Don't try to google answers to these problems! Solutions will be posted later in the week– it's much more fun to work them out yourself.

Problem 1. Once a day, you break a large 4×8 rectangular chocolate bar into its smaller 1×1 pieces, to distribute to your class of 32 students. After some time, you notice that *no matter how* you break up the chocolate bar, it always requires the *same* number of breaks. How many does it take? Why?

Remarks: You can only break the chocolate along its grid lines, and you can only break a *single* piece at a time (the way one might actually break up a chocolate bar). If you can do this problem, consider a chocolate bar of size $m \times n$. Now how many does it take?

Problem 2. You begin with a cube made from 27 smaller cubes glued together (arranged just like a Rubik's cube, although this has nothing to do with this problem). You would like to cut it into its 27 component cubes with a large knife– what is the minimum number of cuts required to do so? How do you know?

Remarks: This is not a trick question. The knife only cuts in a straight line; it does not bend, and you cannot move the pieces while you are making a cut. You may re-arrange, stack, or assemble the pieces however you would like in between each cut, so that you cut more than one piece at the same time. While reading this remark, you should try to convince yourself that you need at *least* 4 cuts.



Problem 3. You have a single piece of pizza, in the shape of a (perfect) equilateral triangle with side length 1 unit. You and your friend would like to divide it equally: what is the *length* of the shortest curve which cuts the triangle into two pieces of equal area?

Remarks: This is not a trick question. The curve does not have to be a straight line, nor does it have to start or end at any of the corners. The *median* of the triangle (in red on the right) is one curve that works; you should convince yourself that the length of this curve is $\sqrt{3}/2$. Can you do better than this?